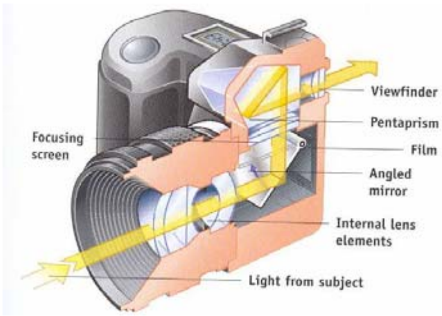
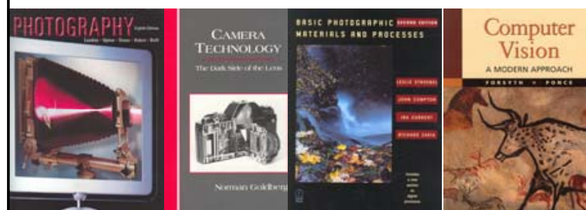
Image Formation



Reading: Chapter 1, Forsyth & Ponce
Optional: Section 2.1, 2.3, Horn.

Reference

- http://en.wikipedia.org/wiki/Lens_(optics)



- The slides use illustrations from these books
- Some of the following slides are editted version of the slides borrowed from Marc Pollefys, Sebastian Thrun, Marc Pollefys, Fredo Durand

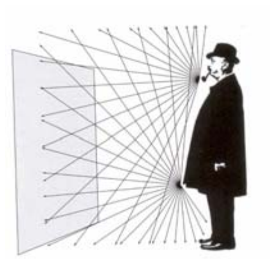
Lecture Overview

- Pinhole optics
- Lenses
- Projections
- Camera

7-year old's question

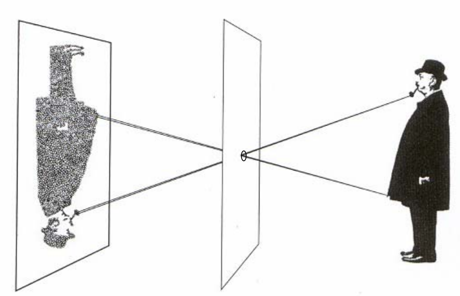
Why is there no image on a white piece of paper?

It receives light from all directions



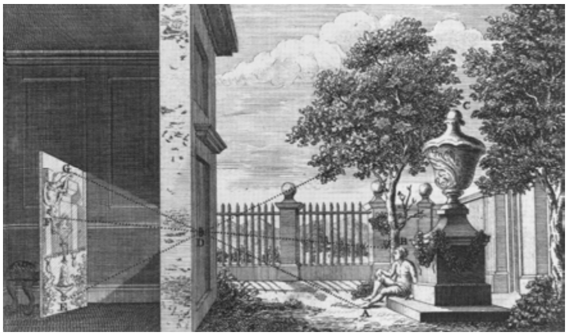
From Photography, London et al.

Pinhole

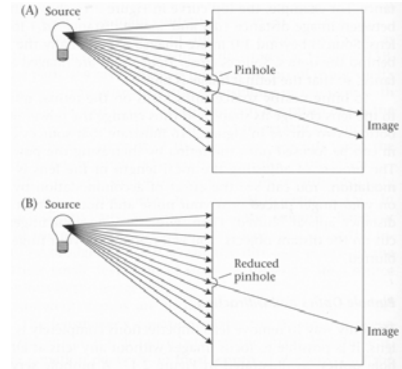


From Photography, London et al.

Camera Obscura

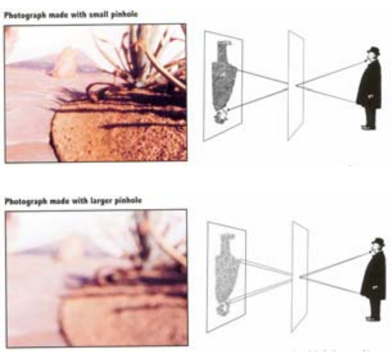


Pinhole Size



Wandell, Foundations of Vision, Sinauer, 1995

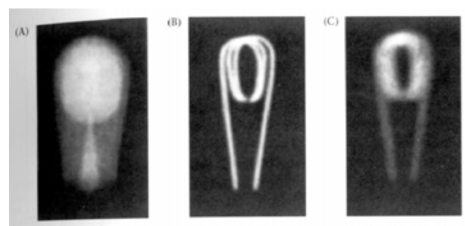
Pinhole size?



From Photography, London et al.

Diffraction limit

- Optimal size for visible light:
  - sqrt(f)/28 (in millimiters) where f is focal length



2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.

From Wandell

Problem with pinhole?

- Not enough light!
  - requires long exposure, may lead to motion blur
- Diffraction limits sharpness

SOLUTION ➔ ***Refraction***

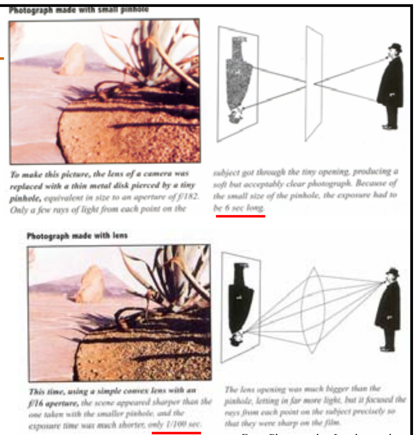
- Refraction is responsible for image formation by lenses and the eye.

Lecture Overview

- Pinhole optics
- ► Lenses
- Projections
- Camera

Lenses

- gather more light!
- But need to be focused


Photograph made with small pinhole

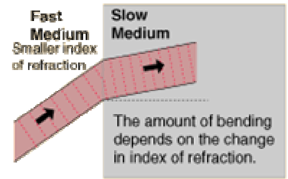
To make this picture, the lens of a camera was replaced with a thin metal disk pierced by a tiny pinhole, equivalent in size to an aperture of f/182. Only a few rays of light from each point on the subject got through the tiny opening, producing a soft but acceptably clear photograph. Because of the small size of the pinhole, the exposure had to be 6 sec long.

Photograph made with lens

This time, using a simple convex lens with an f/16 aperture, the scene appeared sharper than the one taken with the smaller pinhole, and the exposure time was much shorter, only 1/100 sec. The lens opening was much bigger than the pinhole, letting in far more light, but it focused the rays from each point on the subject precisely so that they were sharp on the film.

From Photography, London et al.

Refraction

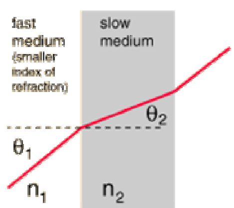


- Refraction is the bending of a wave when it enters a medium where it's speed is different.
- The refraction of light when it passes from a fast medium to a slow medium bends the light ray toward the normal to the boundary between the two media.
- The amount of bending depends on the indices of refraction of the two media and is described quantitatively by Snell's Law.

Snell's Law

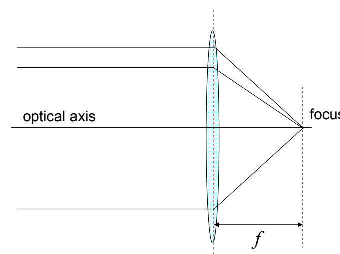
Snell's Law

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1}$$



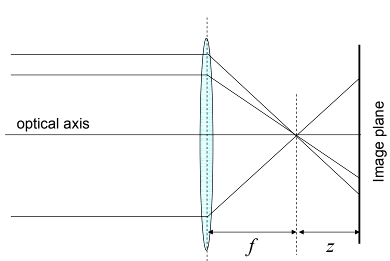
n_i -- is the index of refraction which is defined as the speed of light in vacuum divided by the speed of light in the medium.

Thin Lens



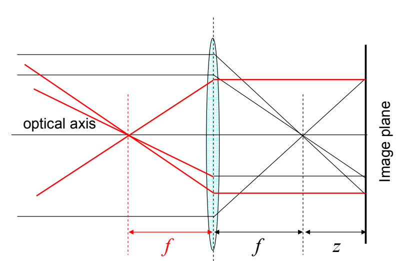
Spherical lense surface: Parallel rays are refracted to single point

Thin Lens: Projection



Spherical lense surface: Parallel rays are refracted to single point

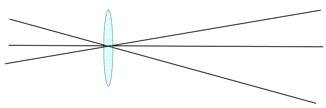
Thin Lens: Projection



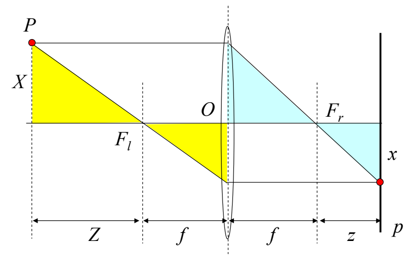
Spherical lense surface: Parallel rays are refracted to single point

Thin Lens: Properties

- Any ray entering a thin lens parallel to the optical axis must go through the focus on other side
- Any ray entering through the focus on one side will be parallel to the optical axis on the other side
- All rays going through the center are not deviated
  - Hence same perspective as pinhole



Thin Lens Model

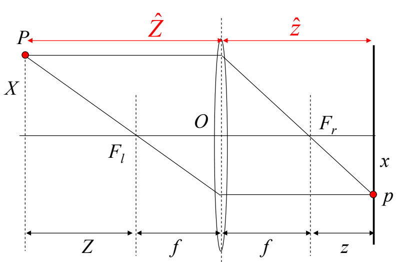


$$\frac{X}{Z} = \frac{x}{f}$$

$$\frac{x}{z} = \frac{X}{f}$$

$$x = \frac{f}{Z} X = \frac{z}{f} X \qquad \frac{f}{Z} = \frac{z}{f} \Rightarrow f^2 = zZ$$

Thin Lens Model



$$zZ = (\hat{Z} - f)(\hat{z} - f) \quad = \quad f^2 = zZ$$

Thin Lens Law Derivation

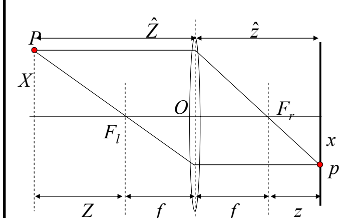
$$Zz = f^2$$

$$(\hat{Z} - f)(\hat{z} - f) = f^2$$

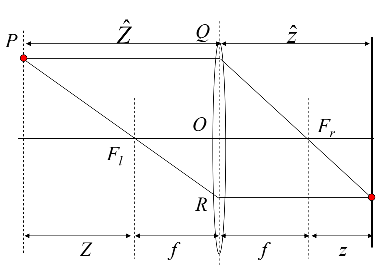
$$\hat{Z}\hat{z} - f(\hat{Z} + \hat{z}) + f^2 = f^2$$

$$\hat{Z}\hat{z} = f(\hat{Z} + \hat{z})$$

$$\frac{1}{f} = \frac{\hat{Z} + \hat{z}}{\hat{Z} \cdot \hat{z}} = \frac{1}{\hat{Z}} + \frac{1}{\hat{z}}$$



The Thin Lens Law

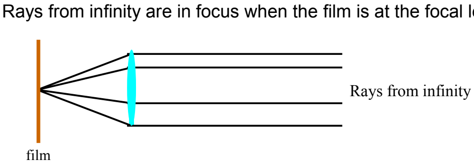


$$f^2 = zZ$$

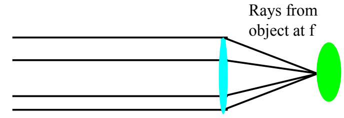
$$\frac{1}{\hat{z}} + \frac{1}{\hat{Z}} = \frac{1}{f}$$

Minimum focusing distance

- Focusing distance –
- Rays from infinity are in focus when the film is at the focal length

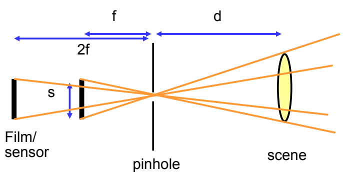


- An object at the focal length requires the film to be at infinity.



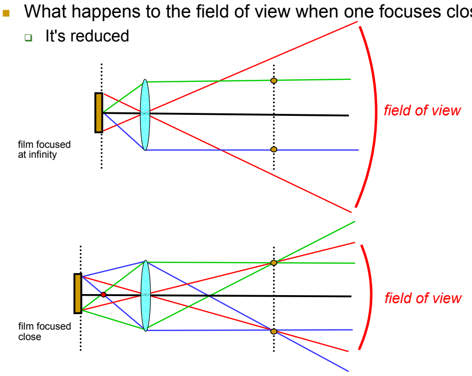
Focal length

- What happens when the focal length is doubled?
  - Projected object size
  - Amount of light gathered



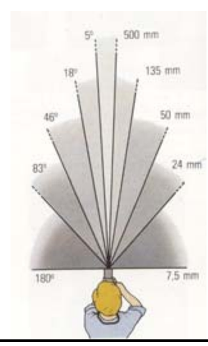
Field of view & focal length

- What happens to the field of view when one focuses closer?
  - It's reduced



Focal length in practice

- Focal length changes subject size



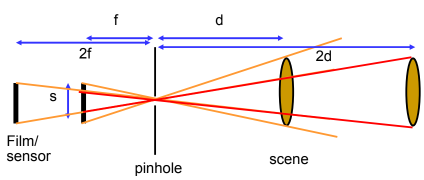
24mm

50mm

135mm

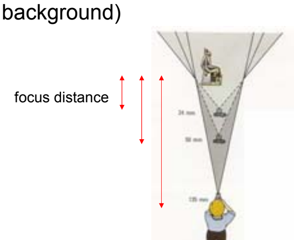
Focal length: pinhole optics

- What happens when the focal length is doubled?
- What happens when the scene is twice as far?
- How do we get the same relative object size when the focal length is doubled?
  - What is the difference then?
    Is it equivalent to get closer and to zoom in?



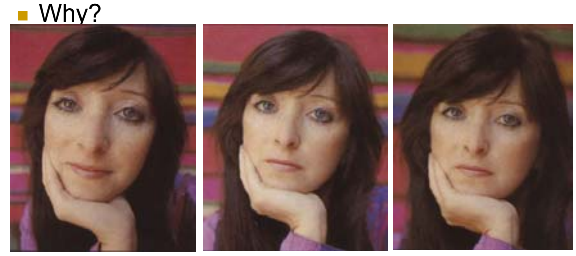
Change in focal length & focus distance

- Same size foreground object by moving the focus distance
- Different perspective (e.g. background)



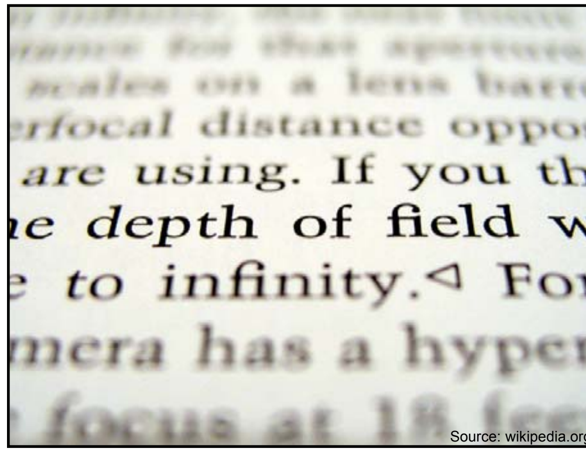
Change in focal length & focus distance

- Portrait: distortion with wide angle
- Why?



Wide angle Standard Telephoto

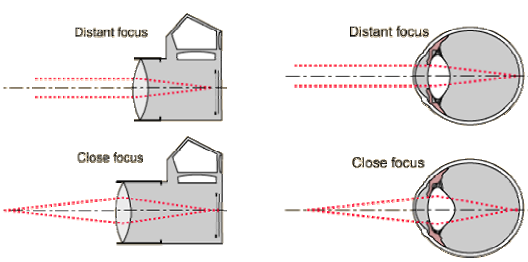
Depth of Field



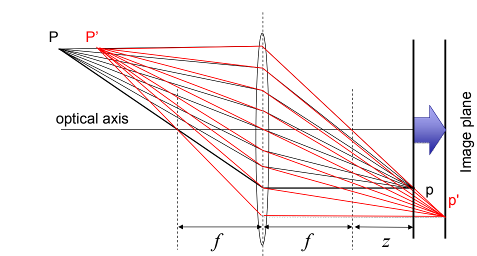
Source: wikipedia.org

To Ways to Change the Depth of Field

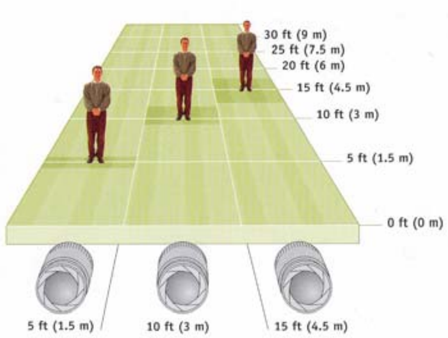
- Change *z* (distance of image plane to lens) - camera
- Deform lens – eye
- Apperture – eye and camera



Thin Lens: Depth of Field



Depends on focusing distance



Human Eye

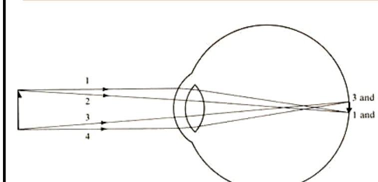


Figure 15.2 Image formation in the eye The light rays bend at the surface of the cornea and upon entering and exiting the lens. An image is formed where the rays from the same part of the object meet. In this case the image lies on the retina, producing clear vision. The distance of the object from the eye is not drawn to scale.

(Theodore D. Ruche and Harry C. Patton, Physiology and Biophysics, 19th ed. Saunders, Philadelphia,1965)

Focusing Through Lens Deformation

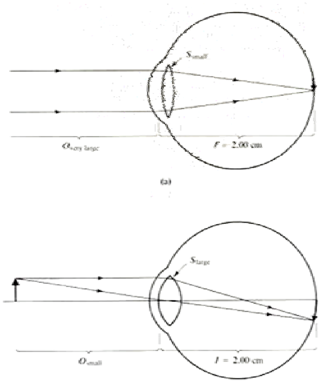
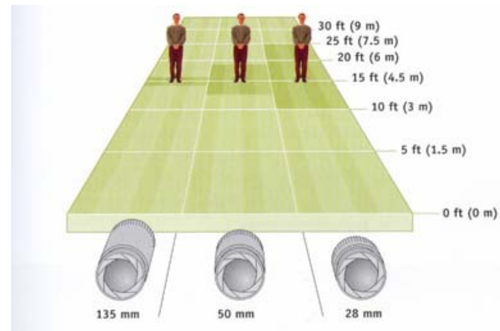
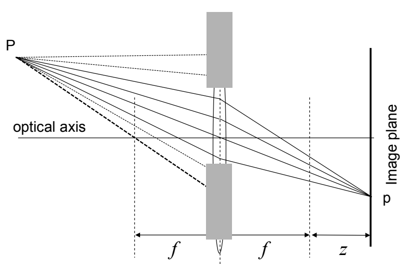


Figure 15.3 Accommodation of lens of eye for **(a)** distance and **(b)** close vision. Note that in **(a)** the two rays are from the same point on the object (not shown). The strength of the eye must increase to converge rays from a close object at the retina.

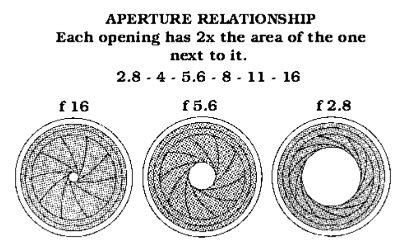
Depends on focal length



Thin Lens: Aperture



Aperture



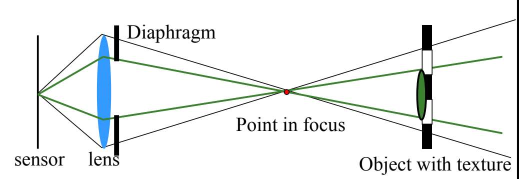
Large Aperture

- Reduces necessary exposure time
- Decreases depth of field

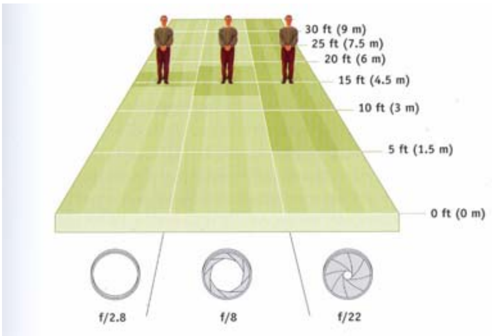
  sometimes desired – most pronounced with telephoto lenses

Depth of field

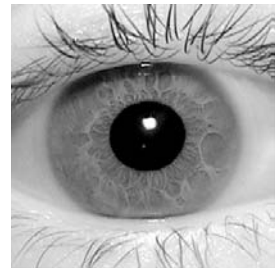
- What happens when we close the aperture by two stop?
  - Aperture diameter is divided by two
  - Depth of field is doubled



DoF Depends on aperture

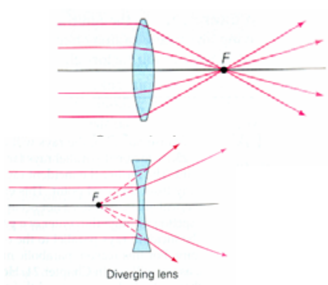


Our Aperture: Iris



Source: www.cl.cam.ac.uk

Convex and concave lenses



http://www.physics.uiowa.edu/~umallik/adventure/light/lenses.gif

Limits of the Thin Lens Model: Aberrations

3 assumptions :

1. all rays from a point are focused onto 1 image point
   - Remember thin lens small angle assumption
2. all image points in a single plane
3. magnification $m = \frac{f'}{z_0}$ is constant

Deviations from this ideal are *aberrations*

Aberrations

2 types :

geometrical : geometry of the lense, small for paraxial rays

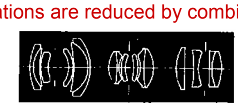
chromatic : refractive index function of wavelength

Marc Pollefeys

Geometrical Aberrations

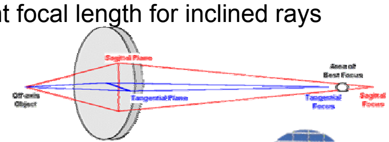
- spherical aberration
- astigmatism
- distortion
- coma

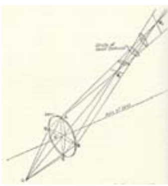
aberrations are reduced by combining lenses

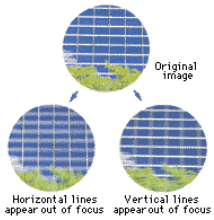


Astigmatism

Different focal length for inclined rays

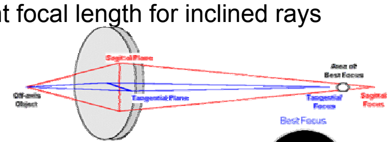


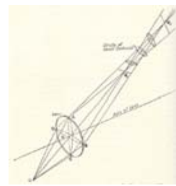


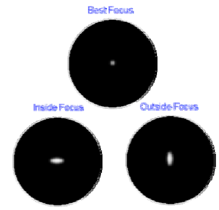


Astigmatism

Different focal length for inclined rays



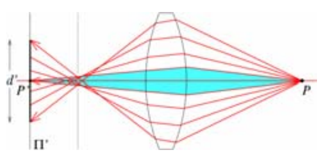




Spherical Aberration

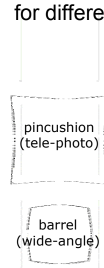
rays parallel to the axis do not converge

outer portions of the lens yield smaller focal lenghts



Distortion

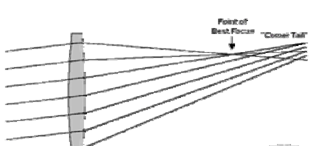
magnification/focal length different for different angles of inclination

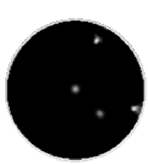


Can be corrected! (if parameters are know)

Coma

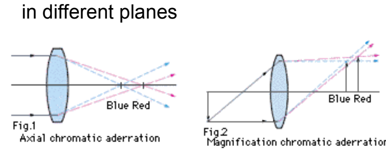
point off the axis depicted as comet shaped blob





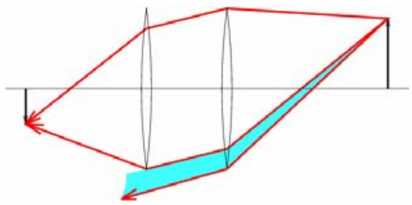
Chromatic Aberration

rays of different wavelengths focused in different planes



cannot be removed completely

Vignetting



Effect: Darkens pixels near the image boundary

Vignetting

Effect: Darkens pixels near the image boundary

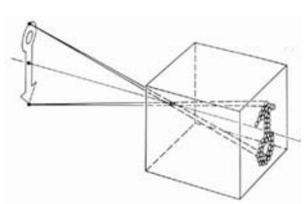
Recap

- **Pinhole camera models the geometry of perspective projection**
- **Lenses make it work in practice**
  - **Refraction: Snell's law**
  - **Thin lens law**
- **Models for lenses**
  - **–**Thin lens, spherical surfaces, first order optics
  - –Thick lens, higher-order optics, vignetting.

Lecture Overview

- Pinhole optics
- Lenses
- ▶ Projections
- Camera

Pinhole camera model

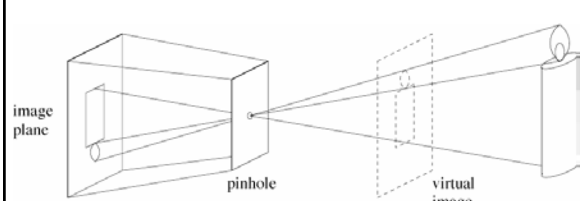


- Pinhole model:
  - Captures **pencil of rays** – all rays through a single point
  - The point is called **Center of Projection (COP)**
  - The image is formed on the **Image Plane**
  - **Effective focal length *f*** is distance from COP to Image Plane

Slide by Steve Seitz

Perspective projection

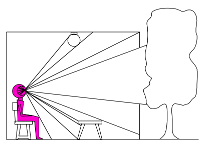
- **Abstract camera model - box with a small hole in it**
- **In an ideal pinhole camera everything is in focus**

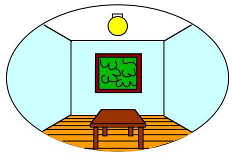


Forsyth&Ponce

Dimensionality Reduction Machine (3D to 2D)

3D world *2D image*

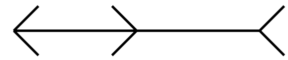




Point of observation

- What have we lost?
  - Angles
  - Distances (lengths)

…but humans adopt!



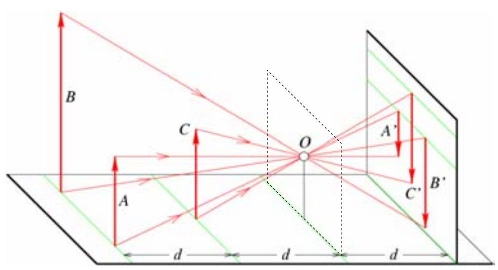
Müller-Lyer Illusion

We don't make measurements in the image plane

http://www.michaelbach.de/ot/sze_muelue/index.html

Distant objects are smaller

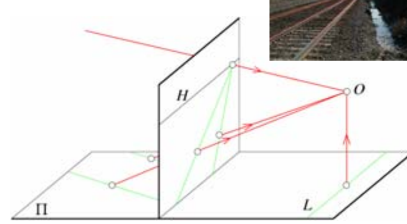
length of B = 2*length of C
length of B' = length of C'



Forsyth&Ponce

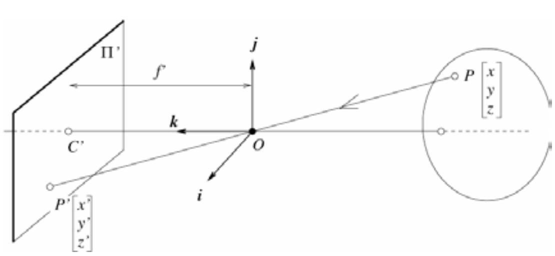
Consequences: Parallel lines meet

- There exist vanishing points

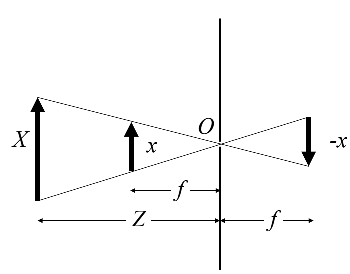


Marc Pollefeys

The equation of projection



Perspective Projection



$$x = -X\frac{f}{Z}$$

The equation of projection

- Cartesian coordinates:
  - We have, by similar triangles, that

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} f\frac{X}{Z} \\ f\frac{Y}{Z} \\ -f \end{bmatrix}$$

  - Ignore the third coordinate, and get

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} f\frac{X}{Z} \\ f\frac{Y}{Z} \end{bmatrix}$$

Homogenous coordinates

- Add an extra coordinate and use an equivalence relation
- for 2D
  - equivalence relation *k*(x,y,z)* is the same as *(x,y,z)*
- for 3D
  - equivalence relation *k*(x,y,z,t)* is the same as *(x,y,z,t)*
- Basic notion
  - Possible to represent points "at infinity"
    - Where parallel lines intersect
    - Where parallel planes intersect
  - Possible to write the action of a perspective camera as a matrix

Homogeneous coordinates

- Is this a linear transformation?
  - no—division by z is nonlinear
- Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

- Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Slide by Steve Seitz

The camera matrix

- Turn previous expression into homogeneous coordinates
  - HC's for 3D point are *(X,Y,Z,t)*
  - HC's for point in image are *(u,v,w)*

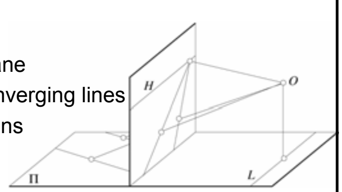
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ t \end{bmatrix}$$

- Position of the point in the image from HC

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z/f \end{pmatrix} \xrightarrow{\text{normalize by } w} \frac{1}{w}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{f}{Z}\begin{pmatrix} X \\ Y \\ Z/f \end{pmatrix} = \begin{pmatrix} fX/Z \\ fY/Z \\ 1 \end{pmatrix}$$

Effect of projection

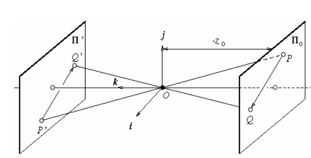
- Points go to points
- Lines go to lines
- Planes go to a half plane
- Parallel lines go to converging lines
- Polygons go to polygons



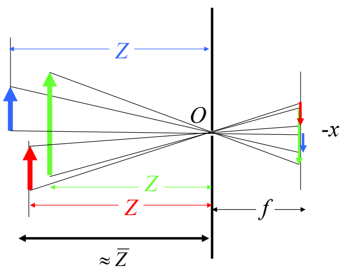
- Degenerate cases:
  - Line through the pinhole go to points
  - Planes through the pinhole go to a line
  - Parallels parallel to the image plane stay parallel
  - Planes parallel to the image plane goes to full planes

Weak perspective

- Issue
  - perspective effects, but not over the scale of individual objects
  - collect points into a group at about the same depth, then divide each point by the depth of its group
  - Adv: easy
  - Disadv: wrong



Weak Perspective Projection



$$x \approx X\frac{f}{\bar{Z}} = const \cdot X$$

Pictorial Comparison

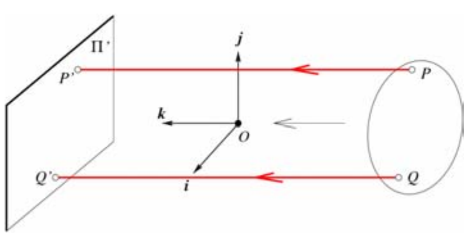
Weak perspective

Perspective

Marc Pollefeys

Orthographic Projection

Telescope projection can be modeled by orthographic projection



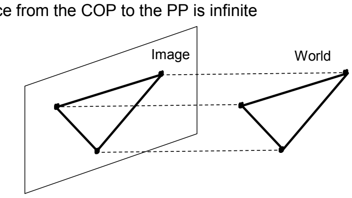
$$\begin{cases} X = x \\ Y = y \end{cases}$$

When the camera is at a (roughly constant) distance from the scene, take m=1.

Marc Pollefeys

Orthographic Projection

- Special case of perspective projection
  - Distance from the COP to the PP is infinite



  - Also called "parallel projection"
  - What's the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \mathbf{1} \end{bmatrix} = \begin{bmatrix} x \\ y \\ \mathbf{1} \end{bmatrix} \Rightarrow (x, y)$$

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Projection Summary:

1. Perspective

$$x = X\frac{f}{Z} \qquad y = Y\frac{f}{Z}$$

2. Weak perspective

$$x = const\ X \qquad y = const\ Y$$

3. Orthographic

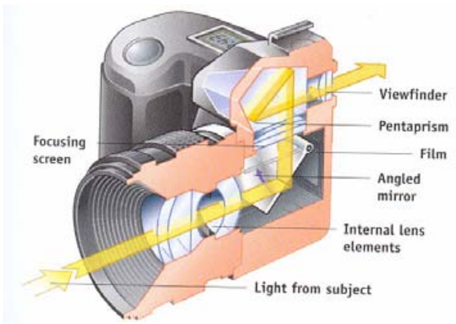
$$x = X \qquad y = Y$$

x, y = image coordinates
X, Y, Z = world coordinates
Z = depth
f = focal length of the camera

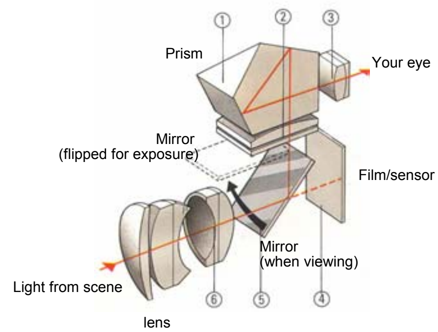
Lecture Overview

- Pinhole optics
- Lenses
- Projections
- ▶ Camera

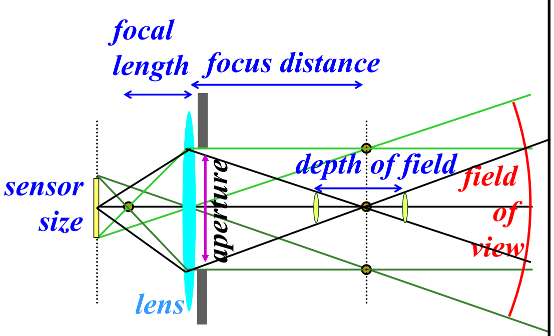
Camera Overview



SLR view finder



Recap: Camera Terminology



Terminology

- Focal length (in mm)
  - Determines the field of view.
    wide angle (<30mm) to telephoto (>100mm)
- Focusing distance
  - Which distance in the scene is sharp
- Depth of field
  - Given tolerance, zone around the focus distance that is sharp
- Aperture (in f number)
  - Ratio of used diameter and focal lens.
    Number under the divider ➔ small number = large aperture
    (e.g. f/2.8 is a large aperture, f/16 is a small aperture)
- Shutter speed (in fraction of a second)
  - Reciprocity relates shutter speed and aperture
- Sensitivity (in ISO)
  - Linear effect on exposure
  - 100 ISO is for bright scenes, ISO 1600 is for dark scenes

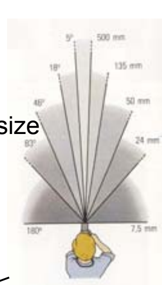
Focal length

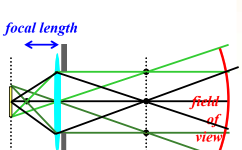
<30mm: wide angle

50mm: standard

>100mm telephoto

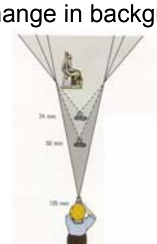
Affected by sensor size
(crop factor)





Change in focal length & focus distance

- Telephoto makes it easier to select background (a small change in viewpoint is a big change in background.

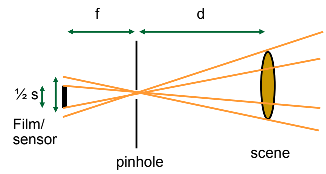


Change in focal length and focus distance

Wide angle Standard Telephoto

Focal length & sensor

- What happens when the film is half the size?
- Application:
  - Real film is 36x24mm
  - On the 20D, the sensor is 22.5 x 15.0 mm
  - Conversion factor on the 20D?
  - On the SD500, it is 1/1.8 " (7.18 x 5.32 mm)
  - What is the 7.7-23.1mm zoom on the SD500?



Sensor size

- Similar to cropping

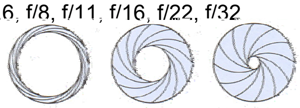
source: canon red book

Exposure

- Get the right amount of light to sensor/film
- Two main parameters:
  - Shutter speed
  - Aperture (area of lens)

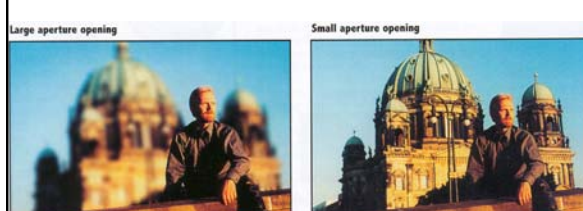
Aperture

- Diameter of the lens opening (controlled by diaphragm)
- Expressed as a fraction of focal length, in f-number
  - f/2.0 on a 50mm means that the aperture is 25mm
  - f/2.0 on a 100mm means that the aperture is 50mm
- Disconcerting: small f number = big aperture
- What happens to the area of the aperture when going from f/2.0 to f/4.0?
- Typical f numbers are
  f/2.0, f/2.8, f/4, f/5.6, f/8, f/11, f/16, f/22, f/32
  - See the pattern?



Main effect of aperture

- Depth of field



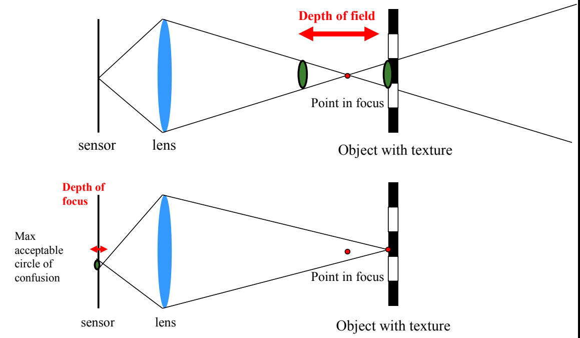
From Photography, London et al.

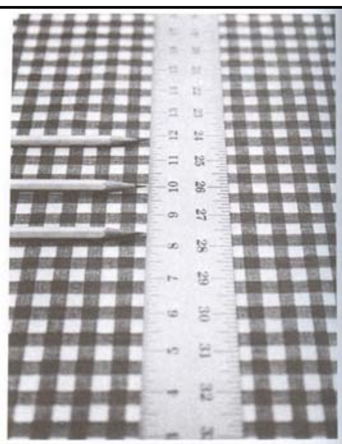
Depth of field

- The bigger the aperture (small f number), the shallower the DoF
  - Just think Gaussian blur: bigger kernel ➔ more blurry
  - This is the advantage of lenses with large maximal aperture: they can blur the background more
- The closer the focus, the smaller the DoF
- Focal length has a more complex effect on DoF
  - Distant background more blurry with telephoto
  - Near the focus plane, depth of field only depends on image size
- Hyperfocal distance:
  - Closest focusing distance for which the depth of field includes infinity
  - The largest depth of field one can achieve.
  - Depends on aperture.

Depth of field and Depth of focus

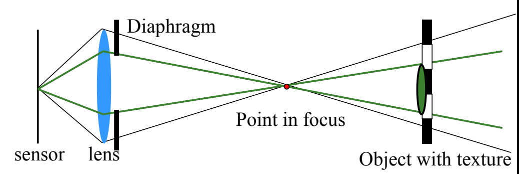
- We allow for some tolerance





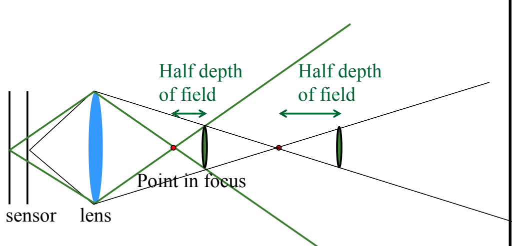
Depth of field

- What happens when we close the aperture by two stop?
  - Aperture diameter is divided by two
  - Depth of field is doubled



Depth of field & focusing distance

- What happens when we divide focusing distance by two?
  - Similar triangles => divided by two as well



Depth of field & focusing distance

- What happens when we divide focusing distance by two?

From Photography, London et al.

Exposure

- Aperture (f number)
  - Expressed as ratio between focal length and aperture diameter: diameter = f / <f number>
  - f/2.0, f/2.8, f/4.0, f/5.6, f/8.0, f/11, f/16 (factor of sqrt (2))
  - Small f number means large aperture
  - Main effect: depth of field
  - A good standard lens has max aperture f/1.8.
    A cheap zoom has max aperture f/3.5
- Shutter speed
  - In fraction of a second
  - 1/30, 1/60, 1/125, 1/250, 1/500 (factor of 2)
  - Main effect: motion blur
  - A human can usually hand-hold up to 1/f seconds, where f is focal length
- Sensitivity
  - Gain applied to sensor
  - In ISO, bigger number, more sensitive (100, 200, 400, 800, 1600)
  - Main effect: sensor noise

Reciprocity between these three numbers:
for a given exposure, one has two degrees of freedom

Recap

- Pinhole is the simplest model of image formation
- Lenses
  - gather more light - refraction
  - but get only one plane focus
  - cannot focus infinitely close
  - focal length determines field of view
  - real lenses have aberrations
- Thin Lens Law
  - can be used to compute where an object will be located in an image given its location in 3D and the focal length
- Projections:
  - Perspective Projection
    - Non-linear projection
    - Pinhole, Camera
  - Weak Perspective Projection – linear
  - Orthographic – models telephoto lens